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A Pseudo-Global Optimization Approach with Application to the Design of Containerships

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Abstract. In this paper, we present a new class of pseudo-global optimization procedures for solving formidable optimization problems in which the objective and/or constraints might be analytically complex and expensive to evaluate, or available only as black-box functions. The proposed approach employs a sequence of polynomial programming approximations that are constructed using the Response Surface Methodology (RSM), and embeds these within a branch-and-bound framework in concert with a suitable global optimization technique. The lower bounds constructed in this process might only be heuristic in nature, and hence, this is called a pseudo-global optimization approach. We develop two such procedures, each employing two alternative branching techniques, and apply these methods to the problem of designing containerships. The model involves five design variables given by the design draft, the depth at side, the speed, the overall length, and the maximum beam. The constraints imposed enforce the balance between the weight and the displacement, a required acceptable length to depth ratio, a restriction on the metacentric height to ensure that the design satisfies the Coast Guard wind heel criterion, a minimum freeboard level as governed by the code of federal regulations (46 CFR 42), and a lower bound on the rolling period to ensure sea-worthiness. The objective function seeks to minimize the required freight rate that is induced by the design in order to recover capital and operating costs, expressed in dollars per metric ton per nautical mile. The model formulation also accommodates various practical issues in improving the representation of the foregoing considerations, and turns out to be highly nonlinear and nonconvex. A practical test case is solved using the proposed methodology, and the results obtained are compared with those derived using a contemporary commercialized design optimization tool. The prescribed solution yields an improved design that translates to an estimated increase in profits of about $18.45 million, and an estimated 27% increase in the return on investment, over the life of the ship.

Key words: Pseudo-Global Optimization, Response surface methodology, Containership design, Branch-and-reduce optimization navigator (BARON), Reformulation-linearization technique (RLT)

1. Introduction

Several engineering and process design problems lead to formidable optimization models that contain analytically complex objective and/or constraints that are expensive to evaluate, or that might be available only as black-box functions. Such problems are also typically highly nonlinear and nonconvex in nature. Because of these challenging features, problems of this type are not amenable to traditional local or global optimization techniques. The new class of *pseudo-global optimization* procedures we develop in this paper are geared toward addressing such problems.

One approach that has frequently been employed to solve these types of complex optimization problems is Response Surface Methodology (RSM). In its traditional form (see Myers (1995), for example), this technique attempts to optimize some black-box or expensive-to-evaluate objective function subject to box-constraints (where other constraints might be incorporated into the objective function via suitable penalty terms). The methodology employs quadratic approximations to the objective function in order to ascertain whether a current solution is a local optimum, or else, to detect a search direction. A suitable step length along this direction leads to a new solution, and the procedure is reiterated. However, this process is intrinsically memoryless. Joshi et al. (1998) describe an enhanced conjugate gradient type of scheme within the context of an RSM-simulation-optimization framework that utilizes previously generated information to compose more effective search directions. Neddermeijer et al. (2000) discuss in detail an RSM framework for optimizing functions that are evaluated via a stochastic simulation model.

Jones et al. (1998) develop a novel RSM-based approach, called the *Efficient Global Optimization* (EGO) method, to optimize black-box functions that are expensive to evaluate. A model function is fit to a set of current points at each stage using maximum likelihood parameter estimators in concert with a nonlinear stochastic process model predictor called DACE (Design and Analysis of Computer Experiments). This model (or a revised form of it based on certain diagnostic goodness-of-fit tests) is used to define a suitable expected improvement utility function that is maximized to determine a new point at which the function is evaluated. The process then reiterates until the expected improvement is less than 1% of the current best objective value.

Another specialized RSM type approach for optimizing complex functions (or penalty functions) over a hyperrectangle is the *Radial Basis Function* (RBF) method due to Gutmann (2001). Here, given a set of points at any stage, a radial basis interpolating function is defined as a weighted sum of a cubic or a thin plate spline function of the Euclidean distances from the given points, plus an affine term. This interpolating function is locally minimized and the result is used to compute a target value based on which an auxiliary function is defined. The minimization of the latter function is simpler than that of the original function, and the resulting solution from executing this step is used to define a new point. This process is continued until either an original objective value close to some goal is attained or some maximum number of iterations have been performed. Gutmann suggests different forms for the radial basis and the auxiliary functions. Some alternative choices of auxiliary functions that tend to be numerically more stable, as well as improved factorizations for implementing this method, are presented in Bjorkman and Holmstrom (2001). Several test cases of industrial and finance problems, including one of designing certain features of a passenger train to minimize the total vehicle mass subject to constraints on ride quality measures, are solved to demonstrate the efficacy of the proposed approach.

It might be worth commenting here that as discussed by Alexandrov et al. (1998), trust region methods for unconstrained local minimization can be adapted to enhance RSM-type strategies. In trust region procedures, at any iteration, based on current (and previous) function and gradient evaluations, an updated second-order Taylor series model is constructed and is optimized over a trust region. Depending on the ratio of the actual to the predicted improvement in objective value, the new solution is either accepted or rejected, and the trust region is either expanded or contracted (or left unaffected). This process continues until a first-order local optimality condition is satisfied. Within this framework, depending on the application, a response surface, or a simulation model, or any other approximation model can be used in lieu of the second-order Taylor series model. A resulting method of this type, as pointed out by Alexandrov et al. (1998), might be more meaningful and have proven local convergence properties related to the original function, in contrast with simpler approximation model solution algorithms adopted frequently in structural optimization contexts, as reviewed for example in Barthelemy and Haftka (1993). Powell (2002) also describes a technique for unconstrained minimization through the use of quadratic models that are fit on an iteratively updated set of interpolation points, along with trust region techniques. To achieve convergence, error bounds on the quadratic model as previously prescribed by Powell (2001) are used, based on the assumption that the objective function has third derivatives bounded by a specified constant.

Jones et al. (1993) propose a variation of a Lipschitzian optimization method called DIRECT that is based on evaluating the objective function value at the centers of a sequence of hypercubes. (Constraints other than the hypercube restrictions are accommodated within the objective function via a linear (Lagrangian) penalty term.) An intrinsically estimated Lipschitz constant is used to guide the selection of the most promising hypercube to explore. Cox et al. (2001) evaluate this procedure in comparison with other traditional search methods in the context of designing high speed civil transport aircraft, and report its relative superiority in solving test cases that exhibit widely separated local minima. Knill et al. (1999) also describe an RSM approach for solving such supersonic aircraft design problems.

Booker et al. (1999) present a *surrogate management framework* for constructing a sequence of approximations for minimizing expensive to evaluate objective functions over simple box constraints. The focus is on constructing surrogates via interpolation approximations using kriging, and optimizing these by applying pattern search algorithms that are proven to converge to stationary points for differentiable functions. An extension of this methodology to handle more general constraints is posed as a future research direction.

Jones (2001) presents an excellent discussion and taxonomy of global optimization methods based on response surfaces. The taxonomy mainly partitions RSM approaches into two-stage procedures wherein a surface is fit first and then a next iterate is found by optimizing an auxiliary function defined by this surface, and one-stage methods that directly use the machinery of RSM to evaluate hypotheses regarding the location of an optimum. Five two-stage methods are discussed based on using quadratic response surfaces, or employing interpolating surfaces via splines or kriging, or conducting a global search based on statistical lower bounding functions, or iterating by either maximizing the probability of detecting a function value that is better than a specified target, or by maximizing the expected improvement upon sampling at the next point. The two one-stage methods discussed use RSM to evaluate the hypothesis regarding the location of where it would be most credible to detect a new iterate that would achieve either a prescribed targeted objective value, or several such values at each iteration. While all these methods are developed for unconstrained (or box-constrained) problems, Schonlan et al. (1997) have extended the two-stage RSM approaches based on maximizing the expected improvement to handle constraints as well. Notwithstanding such contributions, Jones (2001) suggests the extension of RSM-based methods to constrained problems as an open and important challenge for research.

The pseudo-global optimization approach proposed in the present paper is one such technique for solving formidable constrained optimization problems. This is a two-stage RSM approach where first, over a current bounding hyperrectangle in a branch-and-bound framework, a forward stepwise regression process is applied to a feasibility screened, full or fractional factorial experimental design to develop (up to) fifth-order polynomial approximations for the objective and constraint functions. Note that a simple minimization of this polynomial approximating problem can easily miss the global optimum (as illustrated by Jones (2001) for the unconstrained quadratic response surface minimization case). Instead, two alternative approaches are adopted at the subsequent stage. In the first approach, the nonconvex polynomial programming approximating problem is solved to global optimality. (The software package BARON (Branch-and-Reduce Optimization Navigator — see Sahinidis, 1996), or the Reformulation-Linearization Technique (RLT) procedure of Sherali and Tuncbilek (1992, 1997), can be used for this purpose.) The resulting solution is refined by the application of a local search method. The motivation is that the polynomial approximating problem is likely to yield solutions in the vicinity of the true underlying global optimum, and hence, the application of a local search method initiated at such a solution has a greater prospect of detecting such a global optimum. Treating the polynomial approximation solution as a pseudo lower bound, this foregoing analysis is embedded as the node strategy in a branch-and-bound algorithm, using two suitable branching schemes that are designed to induce convergence. As such, because of the assumption on the validity of the heuristic lower bound, we refer to this methodology as a *pseudo-global optimization technique*.

For the second proposed pseudo-global optimization scheme, in lieu of solving the nonconvex polynomial programming approximation to global optimality, we generate a higher dimensional linear programming (LP) relaxation for this polynomial program via a suitable RLT procedure. This LP relaxation value is then taken as a (pseudo) lower bound, and is incorporated within a branch-and-bound framework as for the first method.

As an illustration, we apply this class of methods to a practical test case concerned with the design of containerships. A contemporary ship design tool has been developed by Neu et al. (2000), wherein various aspects of the problem formulation are constructed as modules that are linked together and are used to generate the objective and constraint functions. These modules relate to the geometry, hydrostatics, resistance, propulsion, lightship weight, cargo, total weight, and the economics of the problem. This framework is further linked with a nonlinear optimization tool (*Design Optimization Tools* (DOT) developed by Vanderplaats Research and Development, Inc.), that incorporates various local search algorithms such as the modified method of feasible directions, sequential linear programming, and sequential quadratic programming (see Bazaraa et al. (1993) for a description of these algorithms). We employ this design tool to generate our model, incorporating certain additional modeling improvements as discussed in Section 3 below. We also compare the results obtained by using Neu et al.'s optimization tool to solve the derived problem, versus employing our proposed methodology. In our computational experimentation using a typical practical test case, both our proposed methods found the same solution using two alternative partitioning schemes, where this solution significantly improved upon that prescribed by the commercial design optimization tool employed by Neu et al. (2000).

The remainder of this paper is organized as follows. In Section 2, we present the structure of our proposed class of pseudo-global optimization methods. In Section 3, we provide a conceptual overview of the containership design model formulation, including a discussion on the related literature and the modeling aspects for which we have developed an improved representation. Computational results on applying the proposed pseudo-global optimization methods to solve the containership design problem and comparisons with the incumbent approach of Neu et al. (2000) are given in Section 4. Finally, Section 5 summarizes our contributions, and recommends avenues for further research, as well as extensions to other problems such as the design of warships.

2. Pseudo-Global Optimization Algorithms (PGO1 and PGO2)

Consider an optimization problem stated in the following form.

$$\text{Minimize} \quad f(x) \tag{1a}$$

$$\text{subject to} \quad x \in X \tag{1b}$$

$$x \in \Omega^0 \equiv \left\{x : \ell_j^0 \leqslant x_j \leqslant u_j^0, \ \ \forall j = 1, \dots, n\right\} \tag{1c}$$

where $x \in R^n$ represents the set of decision variables, and where $X \subseteq R^n$ is defined in terms of certain inequality and/or equality constraints. The objective (1a) and these constraint functions might be black-box functions, or some complex analytically defined functions that are expensive to evaluate. We also assume that the indices $j = 1, \ldots, n$ of the variables represent a user-defined ranking that reflects a nonincreasing order of priority among these decision variables. (This priority indexing will play a role in the branching decisions, and we will illustrate using our application in the next section how such a ranking might be inferred in the absence of any related prior information.)

We now propose two pseudo-global optimization approaches (denoted as **PGO1** and **PGO2**) to solve problems of this type. These approaches are based on iteratively using the Response Surface Methodology (RSM), or curve-fitting procedures, in concert with certain global optimization schemes for effectively solving polynomial programming problems (see Sahinidis (1996), and Sherali and Tuncbilek (1992, 1997)), in order to design an overall algorithmic procedure. At each step of this process, given a current hyperrectangle bounding the design variable space, we first construct fifth-order polynomial approximations to the (nonpolynomial) objective and constraint functions. Note that in general, for a function of *n* variables, a polynomial approximation of order *d* will have

$$\binom{n+d}{d}$$

terms, including a constant value. This can be done by using a full or fractional factorial design over the current hypercube, and performing a regression analysis to obtain the coefficients of the fitted polynomial response function (see Myers, 1995). Alternatively, some interpolating technique as advocated by Jones (2001) can be employed to develop these objective and constraint response surfaces.

Accordingly, at some node k in the associated branch-and-bound framework, let

$$\Omega^k \equiv \left\{x : \ell_j^k \leqslant x_j \leqslant u_j^k, \quad \forall j = 1, \ldots, n\right\}$$

denote the corresponding hyperrectangle that defines the current bound restrictions on the decision variables. Hence, the feasible region for the node k subproblem is given by $X \bigcap \Omega^k$. Let the polynomial programming problem generated by replacing the objective and constraint functions with their polynomial response surface approximations over Ω^k be denoted by **PPk**.

In the first pseudo-global optimization approach, abbreviated **PGO1**, we solve the foregoing nonconvex polynomial program PPk to global optimality to yield a solution $\overline{x}^k$ of objective value z_k. (For this purpose, we can employ the software package BARON (Branch-and-Reduce Optimization Navigator — see Sahinidis, 1996), or adopt the RLT-based procedure of Sherali and Tuncbilek (1992, 1997).) The value z_k is *assumed* to represent a lower bound on the node subproblem, although this might not be necessarily true in theory. (For this reason, the overall procedure is called a *pseudo-global optimization method*.) Using the resulting solution $\overline{x}^k$ as a starting solution, we next apply a nonlinear programming search method to the original problem (1) in order to identify a local minimum. The motivation for this is that if the polynomial approximating problem is a sufficiently reliable representation of the true underlying problem over the subregion of concern, the solution $\overline{x}^k$ is likely to be a near-global optimum to the original problem over this region. Hence, refining $\overline{x}^k$ to a local optimum is likely to yield a good quality solution. The resulting solution, x^{k*}, is used to update the incumbent solution x^* of objective value z^*, if necessary. If $z_k \geqslant z^*(1-\varepsilon)$, for some optimality tolerance $0 \leqslant \varepsilon \leqslant 1$ (we used $\varepsilon = 0.01$ in our computations), we fathom this node k. Otherwise, we partition the node k into two subnodes based on splitting the current interval $\left[\ell_q^k, u_q^k\right]$ of a selected variable x_q at a suitable value x_q^k as follows.

We first define

$$x^k \equiv \begin{cases} x^* \text{ if } x_j^* \in \left[\ell_j^k, u_j^k\right] \forall j, \quad \text{with } \; x_j^* \in \left(\ell_j^k, u_j^k\right) \text{ for some } j \\ \overline{x}^k \text{ otherwise.} \end{cases} \tag{2}$$

Next, we find the lowest indexed (highest priority) variable x_q for which $\ell_q^k < x_q^k < u_q^k$. Note that such an x_q must necessarily exist by definition in (2) in case $x^k \equiv x^*$. Furthermore, if $x^k \equiv \overline{x}^k$, then such an x_q is likely to exist because at the feasible vertices of the hyperrectangle Ω^k that are included within our response surface designs, the polynomial approximations tend to be exact, whence node k would likely be fathomed in case $\overline{x}^k$ is a vertex of Ω^k. However, whenever such an x_q does not exist, and if there exists an index j for which

$$\ell_j^k < x_j^* < u_j^k \tag{3a}$$

we let q be the smallest such index. Else, failing this, we select

$$q \in \underset{j=1,\dots,n}{\arg\max} \left\{ \left(u_j^k - \ell_j^k\right) / \left(u_j^0 - \ell_j^0\right) \right\}. \tag{3b}$$

For either of the latter two cases in (3), we re-define

$$x_q^k \equiv \begin{cases} 0.1\ell_q^k + 0.9u_q^k & \text{if } \; \overline{x}_q^k = u_q^k \\ 0.9\ell_q^k + 0.1u_q^k & \text{if } \; \overline{x}_q^k = \ell_q^k. \end{cases} \tag{4}$$

Accordingly, we now create two subnodes for node k by replacing the corresponding bounding restrictions $\ell_q^k \leqslant x_q \leqslant u_q^k$ for the variable x_q by

$$\ell_q^k \leqslant x_q \leqslant x_q^k, \text{ and } x_q^k \leqslant x_q \leqslant u_q^k \tag{5}$$

in the two respective successor subproblems. Furthermore, to induce convergence using standard arguments (see Horst and Tuy, 1993), we periodically select q via (3b) along any branch (say, every 10 partitions or so), and bisect the current interval of x_q to obtain the two subnodes. Each of the resulting two subproblems in any case is then analyzed similar to the parent node k. The motivation for this node partitioning or branching rule, abbreviated **BR1**, is that the approximations tend to be exact at the points defined by interval end-points of the variables. Hence, for a node subproblem, if the incumbent solution lies in the corresponding hyperrectangle, we attempt to generate subnodes whose end-points would match with the incumbent solution values, at least with respect to the higher priority variables. The motivation for (4) is to perturb the current relaxation solution away from the vertex of Ω^k with respect to a critical index so as to encourage either detecting an improved incumbent, or fathoming this solution. Otherwise, the (pseudo) lower bounding mechanism is tightened by creating end-points at the corresponding approximating problem's solution with respect to the higher priority variables (via the second case in (2)).

In this overall process, we also maintain a list of active nodes L for which the (pseudo) lower bound is less than $z^*(1-\varepsilon)$. At any stage, we *extract* (i.e., select and remove) a node k from L that has the least lower bound. If $L = \emptyset$, or if we have encountered a total of some $N_{\max}$ nodes, we terminate the overall process, and prescribe the current incumbent solution for implementation. (We recommend $N_{\max} \geqslant 2^{n'}$, if reasonable, where n' is the number of variables for which the incumbent solution value at the root-node is different from the variable bounds, so that this might at least provide the opportunity to explore the $2^{n'}$ subproblems based on a combination of subintervals, each split at the corresponding variables' non-end-point interval values.)

The second node partitioning or branching rule that we propose, abbreviated **BR2**, adopts the following procedure. We first define

$$x^k \equiv \begin{cases} x^* \text{ if } x_j^* \in \left[\ell_j^k, u_j^k\right] \ \forall j, \text{ with } x_j^* \in \left(\ell_j^k, u_j^k\right) \text{ for some } j \\ \overline{x}^k \text{ otherwise, if } \overline{x}_j^k \in \left(\ell_j^k, u_j^k\right) \text{ for some } j \\ \hat{x}^k \text{ otherwise, where } \hat{x}_j^k = 0.75u_j^k + 0.25\ell_j^k \quad \forall j, \end{cases}$$

and let

$b_j^k \equiv$ number of times that x_j has been selected as the branching variable in the path from the root-node to the node k in the enumeration tree thus far, $\forall j$.

The branching variable index q is then selected as

$$q \in \arg \operatorname{lex} \min \left\{\left(b_j^k, j\right) : \ell_j^k < x_j^k < u_j^k, \ j = 1, \ldots, n\right\}, \tag{6}$$

and accordingly, we branch as before via the dichotomous restrictions (5). Again, as in BR1, we periodically select q according to (3b) along any branch, and bisect the current interval of x_q to obtain the two subnodes. The motivation for the modification implemented in BR2 as embodied by (6) is that it avoids selecting the same branching variable if another viable choice exists that has not been considered (or has not been considered as many times) thus far along the chain from the root-node to node k.

The second proposed pseudo-global optimization procedure, PGO**2,** follows the same overall scheme as for PGO1, with one key difference. For any node $k \geqslant 0$, having formulated the polynomial programming approximation PPk, in lieu of solving this polynomial program to optimality as in PGO1, we instead construct an RLT-based tight linear programming (LP) relaxation RLT-PPk, say, for the problem PPk (see Sherali and Tuncbilek, 1992, 1997), and solve this LP relaxation to derive the solution $\overline{x}^k$ of objective value z_k. The remainder of this procedure follows the schema of PGO1 identically. Note again that as shown in Sherali and Tuncbilek (1992), the RLT relaxation value matches that of the original polynomial program at the vertices of Ω^k, and therefore, the motivation for the branching index selection rules BR1 and BR2 remains the same. The key difference is that the (pseudo) lower bound is now being computed via the solution of a *linear* program RLT-PPk, as opposed to a *polynomial* program PPk. The motivation here is that if RLT-PPk is a tight relaxation of PPk, (which the results in Sherali and Tuncbilek (1997) anticipate to be the case), then we might save on computational effort without impairing the quality of the solution derived at termination of the proposed algorithm.

As far as the finite convergence of these pseudo-global optimization procedures PGO1 and PGO2 for any given tolerance $\varepsilon > 0$ is concerned (even with $N_{\max} = \infty$), this follows from the periodic interval bisection in the prescribed branching schemes and the standard convergence arguments expounded in Horst and Tuy (1993) and Sherali and Tuncbilek (1992). However, except under certain additional conditions regarding the structure of the problem and the algorithmic process employed, we cannot guarantee that this convergence will occur to a true (ε-) optimal solution. For example, such a guarantee is assured by the development in Sherali and Wang (2001) if the objective and constraints are factorable functions, and if the polynomial approximations employed are interpolating polynomials, such as Chebyshev polynomials, that can be shifted by some known error bounds. In such cases, the polynomial programs PPk are true lower bounding relaxations, and the branching mechanism employed will induce convergence to a global optimum (see Sherali and Wang, 2001). In a similar spirit, Cox and John (1997) discuss how "statistical lower bounds" can be computed in the context of unconstrained optimization by exploiting the standard error measure that is available when using kriging interpolation techniques. Assuming these errors to reflect true discrepancies between the actual function and the fitted response, a convergent global optimization algorithm could be designed. We recommend the exploration of such methods that would guarantee convergence to a global optimum for various classes of problems of type (1) for future research.

3. Application to a Containership Design Problem

Ship design has traditionally been an iterative process in which different aspects of the problem pertaining to power, strength, stability, weight, and space balance have been performed in sequence to arrive at a variety of feasible solutions. This iterative process of working from mission requirements to a detailed design can be conceived as moving along a Design Spiral (see Taggart (1980)), in which the steps progress from a basic initial design to a final prescribed contract plan via a series of parametric studies that attempt to determine the most economical design. Although the design spiral approach has been refined considerably with the advent of various computer programs, it lacks formalism from a modeling and optimization perspective. This has prompted several researchers to develop optimization approaches that would facilitate an effective synthesis of all the required design specifications, and would provide a useful mechanism for exploring competitive designs.

Chryssosstomidis (1967) proposes an optimization approach to containership design in which the carrying capacity (number of containers) and the speed are fixed during the design process. The measure of merit used attempts to minimize the annualized total cost. A random search optimization technique is employed, that computes values of the objective function for thousands of sample designs satisfying a given set of owner's requirements, each corresponding to a set of values of the independent variables as determined by an exponential random search transformation. Erichsen (1971) presents a mathematical model for containership design optimization that considers operating costs incurred at container port terminals as well as for land transportation, in addition to the annualized ship cost. Here again, the carrying capacity (number of containers) and the speed are fixed during the design process, and results are obtained and compared by using a direct search technique and by employing geometric programming techniques. The latter methodology is shown to produce competitive designs. Keane et al. (1991) describe an integrated computational approach to ship concept design that incorporates accepted naval architectural tools, a sophisticated database handler, and several optimization procedures. The objective used is to minimize resistance, which is calculated by the Holtrop–Mennen regression method (see Holtrop and Mennen (1984)). Sen (1992) advocates Multiple Criteria Decision Making (MCDM) as a more effective approach to marine design, and accordingly, proposes a goal programming model. Ray and Sha (1994) argue that in this model, the identification of the weights associated with the different goals is difficult since the goals are all interrelated. They propose an objective function given by a weighted average of the building cost, power, and steel weight of the ship, and apply local search techniques to solve the problem, while again treating the number of containers and the speed as fixed parameters in the design process. In a follow-on study, Ray et al. (1995) explore a combination of simulated annealing and multistart nonlinear search methods. A variety of local solutions differing significantly in objective value are derived, thereby exhibiting the nonconvexity of the design problem.

In most of these methods, the hull form geometry of the ship is assumed to be fixed to a scale-factor, or is taken as some convex combination of standard hull forms. Peri et al. (2001) describe a discretized shape optimization approach

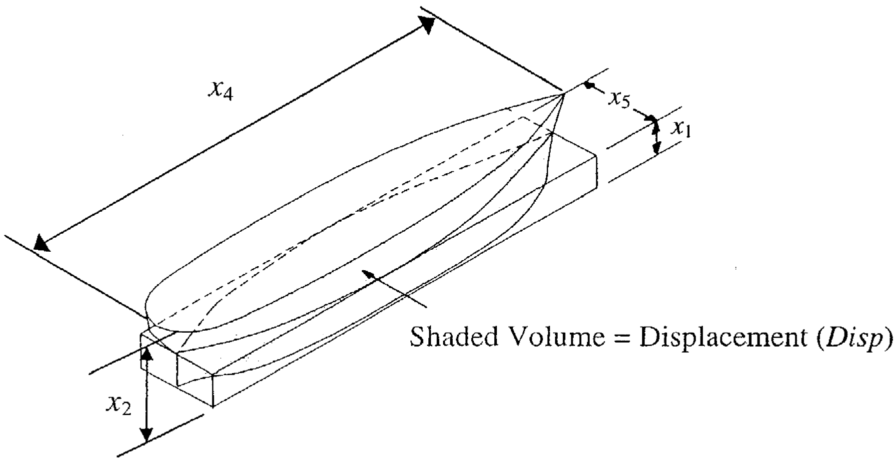


Figure 1. Principal dimensions of the ship

for designing the hull of tanker ships in order to minimize a combination of the resistance of the ship and the amplitude of the wave patterns produced at a specified design speed. Steepest descent, conjugate gradient, and sequential quadratic programming local search methods are tested for solving the generated model. Two new optimal hull geometries, in addition to the conventional bulbous bow design, are identified.

Valorani et al. (2000) investigate more effective strategies for the design of optimal ship hulls, employing sensitivity analysis and adjoint equations based procedures. Shape optimization of the bulbous bow of a hull is used as a test case to compare results obtained by using these enhanced strategies to those obtained via algorithms based on finite differences for gradient computation, demonstrating an estimated 33% CPU time savings for identifying the same optimal design for all three methods. For additional discussion on designing optimal ship hulls, we refer the reader to Tahara et al. (2000).

We now describe a model formulation for the containership design problem that we use as a case study to illustrate the application of our proposed pseudo-global optimization procedures. As mentioned in Section 1, the containership design package developed by Neu et al. (2000) encapsulates the essential aspects of the problem formulation via a set of modules that relate to the geometry, hydrostatics, resistance, propulsion, lightship weight, cargo, total weight, and economics of the problem. These modules perform various computations related to the structural and physical properties, performance characteristics, and economic aspects of the ship, given a set of parameters and design decision variable values. Rather than reproduce the different formulae in terms of the design decision variables that govern these calculations, we shall focus in this section on the basic underlying concepts and the essential naval architectural terms and definitions that are central in constructing the model formulation, including comments on the model enhancements that have been incorporated within the present study. A full detailed statement of the resulting objective and constraint functions (some of which run into hundreds of terms) can be obtained from the appendices of Ganesan (2001) as indicated below. When referring to these appendices, the reader is cautioned to match the variables defined therein with those designated below, where the latter have been reindexed in order of importance as prescribed by our solution methodology.

The *design decision variables* for the problem, prioritized according to their relative importance in the model, are as follows (see Figure 1 for an illustration of the related physical dimensional variables).

1. **Design Draft (in meters),** x_1: *Draft* is defined as the depth of the ship below the waterline, measured vertically to the lowest part of the hull, propellers, or some other reference point. *Design Draft* is defined as the draft under a full-load condition, and is more convenient to use since it frequently occurs in the computations performed within the design process.
2. **Depth at side (in meters),** x_2: *Depth at side* is defined as the molded distance between the ship's baseline and the underside of the deck plating for the uppermost continuous deck, measured at the side of the ship.
3. **Speed (in knots),** x_3: The *service speed* is defined as the predicted average speed at which the ship (at design draft immersion) is expected to operate over its entire life at sea. This prediction takes into account such factors as the environment, fouling, corrosion, and any other aspects that tend to reduce a ship's speed. (*Knot* is a unit of speed, equaling one nautical mile per hour; the international nautical mile is 1852 m.)
4. **Overall Length (in meters),** x_4: The *overall length* is defined as the extreme length of a ship measured from the foremost point of the stem to the aftermost (lattermost) part of the stern.
5. **Maximum Beam (in meters),** x_5: The *maximum beam* is defined as the maximum molded width of the ship measured to the outside of the hull frame angle of channel, but inside of the shell plating.

In addition to the foregoing dimensional design variables, another fundamental characteristic that determines the overall shape of the ship is the *hull form*. Of particular interest are the geometric properties of the form of the hull that would typically be immersed in the water under normal operating conditions. These characteristics are also referred to as *hydrostatic properties* because they pertain to the underwater form of the hull. The hull geometry is typically ascertained by traditional naval architectural integration procedures. In the containership design package developed by Neu et al. (2000), this hull geometry is captured via a user-selected weighted average of two or three basis hull forms, where the latter are described in terms of certain net point vectors that depend on the overall dimensions of the ship. While it is possible to include such weights attached to certain basis hull forms as decision variables within the model, users alternatively select such weights to determine the geometrical shape of the hull, but let the overall dimensions of the ship dictate its actual ultimate size and form. We have adopted the latter approach in the present study.

There are five structural constraints that are imposed on the design problem, in addition to suitable lower and upper bounding restrictions on the decision variables. These constraints are described in turn below.

3.1. BALANCE BETWEEN THE WEIGHT AND THE DISPLACEMENT

According to Archimedes' floatation principle, we must necessarily equate the displacement (i.e., the weight of the displaced water assuming a certain sea-water density) with the weight of the ship. The normal design practice is to enforce this mandatory restriction under full-load conditions. Hence, the displacement is directly related to the draft design variable (x_1) and the hull shape (see Figure 1). Note that by specifically including the draft as a decision variable in our model and explicitly enforcing this constraint, we avoid the implicit internal loop employed in previous design approaches where the draft is *iteratively computed* based on the other design and weight characteristics using Archimedes' Principle, and is then required to lie within suitable bounds. Our modified approach makes the model more precise and simplifies the optimization process. In this context, the *displacement term* is computed via numerical integration using the hydrostatic curves (see Neu et al. (2000)).

As far as the *weight term* within this equality constraint is concerned, this value is computed under full-load conditions based on the essential dimensions and hull form of the ship, and its carrying capacity. The weight and cargo modules of Neu et al. (2000) are used for this purpose, where the former computes the lightship, fuel, and other miscellaneous weights, while the latter examines the carrying capacity of the ship in terms of twenty-foot equivalent units (TEUs) of cargo containers that can be accommodated, times the estimated weight per container unit. The *lightship weight* is comprised of the hull steel weight, the outfit and the hull engineering weight, and the machinery weight. The *fuel weight* depends on the engine horsepower and efficiency characteristics, the desired range of the ship as defined by the user, and the design speed (x_3). Other *miscellaneous weights* include those pertaining to the crew and their provisions, freshwater, and the lube-oil for the diesel.

In all previous attempts to formulate the containership design problem, although the principal dimensions of the ship are treated as design variables, the carrying capacity (in terms of the number of containers) is not modeled as a function of these design variables, but is taken as a fixed estimate. However, the principal dimensions of the ship strongly influence its carrying capacity. Therefore, in order to improve the accuracy of the model, we include an explicit treatment of this issue as follows.

(a) The number of containers below deck is expressed as a function of the length, the beam, and the depth of the ship. This has been done by discretizing the space available for container stowage in the lengthwise, the beamwise, and the depthwise directions, and employing a *stowage factor* to calculate the total volume available for holding containers. This stowage factor accounts for the geometry of the hull form and the space occupied by the container cell guides, and depends on the ratio of the displacement volume to that of the hyperrectangle that envelopes the immersed hull form as depicted in Figure 1 (this ratio is called the **block coefficient**).

(b) The number of containers above deck is likewise expressed as a function of the length and the beam of the ship, by using a suitable stowage factor to account for the number of tiers above deck and the geometry of the available surface area. Also, data from Panamax and post Panamax ships indicate that the number of tiers above deck is not fixed, but is a function of the beam. Moreover, it need not be integral because of the use of partial tiers to satisfy visibility requirements. A regression analysis of this data was used to derive an expression for the number of tiers as a function of the ship dimensions.

The overall derived weight expression in terms of the design decision variables was validated against data obtained for seven ships and was observed to match this measured data quite accurately, with an error tolerance having an average of 7.14%, and ranging from 0.4 to 15.2% (see Table 3.1 in Ganesan (2001)).

A complete mathematical form of this displacement-weight balance equality constraint is given in Appendix B of Ganesan (2001).

3.2. LENGTH TO DEPTH RATIO

For the lightship weight formulation to be meaningful, there is a mandatory lower bound on the length to depth ratio that must be satisfied. This is represented as follows:

$$x_4 - 8.3x_2 \geqslant 0. \tag{7}$$

3.3. RESTRICTION ON THE METACENTRIC HEIGHT TO ENFORCE THE COAST GUARD WIND HEEL CRITERION

The center of buoyancy of a listed (or tilted) ship is not on the vertical centerline plane. The intersection of a vertical line drawn through the center of buoyancy of a slightly listed ship (that is tilted by an angle φ according to Coast Guard specifications), intersects the centerline plane at a point called the *transverse metacenter* (see Figure 2). The *transverse metacentric height* is defined as the distance from the transverse metacenter to the center of gravity of a ship, and is given as

$$GM = KM - KG \tag{8}$$

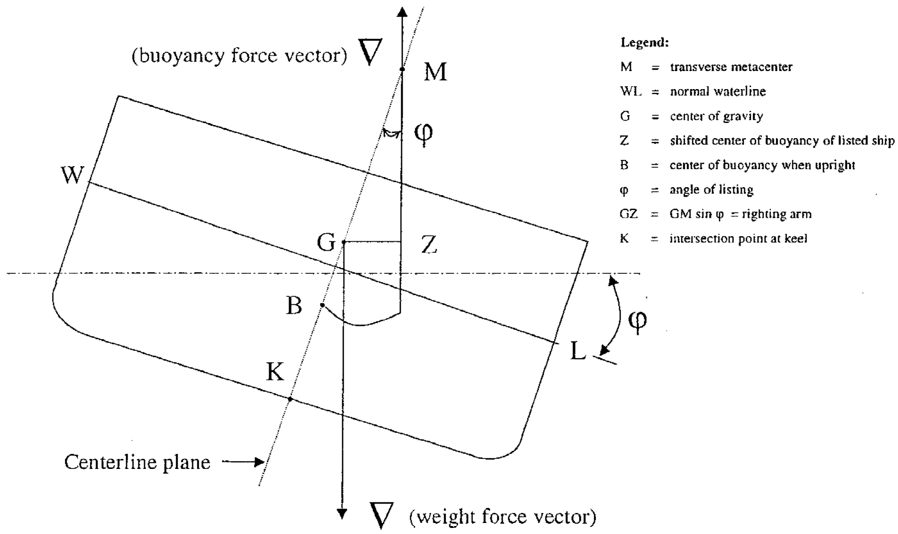


Figure 2. The Metacenter and the Metacentric Height. A complete mathematical form of this constraint is given in Appendix C of Ganesan (2001).

where *KM* is the height of the *transverse metacenter above keel* as computed by the sum of the height of the center of buoyancy above keel (*KB*) plus the *transverse metacentric radius* (*BM*), and where *KG* is the vertical distance between the keel and the center of gravity of the ship (see Figure 2). For small angles of tilt (7–10°), the point *M* remains practically stationary with respect to the ship. Note that *KG* is a function of the design variables as derived by the hydrostatic module of Neu et al. (2000). Also, observe from Figure 2 that the resultant weight and buoyancy force vectors are separated by $GZ \equiv GM \sin\varphi$, which is called the *righting arm*. If the center of gravity is below the metacenter, the vessel is **stable**, i.e., it retains the ability to return to her normal upright position when heeled by the action of waves, wind, or navigational maneuvers. In this case, the righting arm is considered positive, and the net moment of the weight and the buoyancy about GZ tends to rotate the ship to an upright position. To ensure a sufficient degree of stability even under relatively large angles of tilt, the Coast Guard (see Coast Guard, 1997) requires the metacentric height to satisfy a lower bounding inequality given by

$$GM \geqslant \frac{P \times LA \times H}{Disp \times \tan(\theta)} \tag{9}$$

where, referring to Figure 1 for the ship dimensional notation, *Disp* = displacement in cubic meters as described above; θ =angle subtended at the center of the ship by the height above the waterline under full-load condition (i.e., by the difference between the depth and the draft), so that $\tan(\theta) = 2(x_2 - x_1)/x_5$; $P = 0.036 + (Lbp/1309)^2$ metric tons/meter^2, where *Lbp* is the length between the vertical perpendiculars at the fore-side of the stern and the aft-side of the ship at the rudder post (or at the sternpost or at the rudder stock, depending on the ship);

LA = projected lateral area of the portion of the vessel and the deck cargo that are above the waterline, so that $LA \equiv x_4\,[(x_2 - x_1) + 2.4 \times Ntd]$, where *Ntd* is the number of tiers on deck and 2.4 is its height conversion factor (see Figure 1); H = vertical distance from the center of *LA* to the center of the underwater lateral area or to the mean draft point, so that $H = 0.5\,\{(x_2 - x_1) + 2.4 \times Ntd\} + 0.5x_1$.

3.4. MINIMUM FREEBOARD RESTRICTION

The *freeboard* of a ship is defined as the distance from the waterline to the upper surface of the deck measured at side, and is given by the difference $(x_2 - x_1)$ (see Figure 1). In order to ensure sea-worthiness, it is required that the freeboard satisfy the following lower-bounding inequality, which together with (9), assures an adequate righting moment and stability for the ship:

$$(x_2 - x_1) \geqslant [\text{Freeboard\_min}]\rho_1 + \rho_2. \tag{10}$$

Here, Freeboard_min is given by the following expression obtained by fitting a curve using the method of least squares through the 114 points given in the freeboard tables in Taggart (1980), as governed by the code of federal regulations for freeboard (46 CFR 42), and ρ_1 and ρ_2 are variable correction factors as described below.

$$\text{Freeboard\_min} = 0.025633 \times x_4^{0.9146}. \tag{11a}$$

The multiplicative correction factor ρ_1 depends on the **block coefficient** C_b, which is defined as the ratio of the immersed volume of the hull (*Disp*) and the volume $(x_4x_5x_1)$ of the rectangular block defined by the length, beam, and draft of the ship (see Figure 1). This factor corrects for the situation when C_b might be too high (relatively close to a rectangular immersed section), and is given by the following continuous function

$$\rho_1 = \begin{cases} \dfrac{C_b + 0.68}{1.36} & \text{if } C_b > 0.68 \\ 1 \text{ otherwise}. \end{cases} \tag{11b}$$

The additive factor ρ_2 likewise corrects for a too low length-to-depth ratio (x_4/x_2), and is given by the following continuous function

$$\rho_2 = \begin{cases} 0.25\left(x_2 - \dfrac{x_4}{15}\right) & \text{if } \dfrac{x_4}{x_2} < 15 \\ 0 \;\text{ otherwise}. \end{cases} \tag{11c}$$

Note that by the nature of the prescribed solution methodology, the conditional forms of the factors ρ_1 and ρ_2 can be directly accommodated within the algorithmic computations, and therefore, we do not need to mathematically model this structure explicitly using binary variables, for example.

3.5. ROLLING PERIOD CRITERION

Another factor that governs the sea-worthiness of the ship from the viewpoint of the physical comfort of the occupants with respect to sea-sickness is to ensure that the *rolling-period* is not too low, i.e., the ship does not rock sideways about its keel with too great an oscillation frequency. This design constraint is represented as rolling-period $\geqslant$ rolling-period_min, where an expression for the rolling-period in terms of the vertical center of gravity KG (meters) and the metacentric height GM (meters) as defined above (see Figure 2) is given by (see Ni (1998)):

$$\text{rolling-period} = 0.58\sqrt{\left[x_5^2 + (4 \times KG^2)\right] / \left|GM\right|} \text{ seconds,}$$

and where rolling-period_min is defined by the user (in this work it is taken as 15 s). This constraint can be restated as follows:

$$x_5^2 + (4 \times KG^2) - 668.46 \times \left|GM\right| \geqslant 0. \tag{12}$$

A detailed mathematical form of this restriction is given in Appendix D of Ganesan (2001).

3.6. LOWER-UPPER BOUNDING CONSTRAINTS

The ship design variables are typically required to be bounded within some plausible, desired, user-defined limits. In the present study, these imposed bounds are as follows:

$$\begin{array}{ll} \text{Draft:} & 6 \leqslant x_1 \leqslant 11 \text{ m} \\ \text{Depth:} & 12 \leqslant x_2 \leqslant 25 \text{ m} \\ \text{Speed:} & 4 \leqslant x_3 \leqslant 35 \text{ knots} \\ \text{Length:} & 100 \leqslant x_4 \leqslant 300 \text{ m} \\ \text{Beam:} & 20 \leqslant x_5 \leqslant 43 \text{ m.} \end{array} \tag{13}$$

3.7. OBJECTIVE FUNCTION

Containerships, being commercial transportation vessels, are typically designed for generating maximal profits. In our design approach, we analyze the problem from the ship owner's perspective and assume a continuous sufficient demand in the market for the cargo being transported. The objective function we adopt herein, based on standard practice, seeks to minimize the *required freight rate* (**RFR**), expressed in dollars per metric ton per nautical mile, which is simply the amount the owner must charge the customer in order to break-even. A more precise definition from Schneekluth (1987) is: “The required freight for a given rate of utilization produces net revenues that exactly cover the operating costs inclusive of calculated interest on the invested capital". An expression for this function is given by

$$\text{RFR} = \frac{\text{AC}}{\text{NC} \times \text{Wcargo} \times \text{Dst}} \tag{14}$$

where AC = annualized total cost in dollars, NT = number of round-trips made by the ship annually, Wcargo = average weight of the cargo per trip in metric tons, and Dst = average distance per round-trip in nautical miles.

Note that if customers pay an actual charge-rate of CR per metric ton transported per nautical mile, the profit margin per metric ton per nautical mile is given by (CR-RFR), which is in effect being maximized in the present context.

The annualized total cost AC includes the building material and labor costs that are weight dependent, and are annualized using a suitable capital recovery factor, plus annual fuel costs and annual operating costs that are comprised of wages, stores and supplies, insurance, maintenance and repair, port expenses, and cargo handling costs. The weight and economic module of Neu et al. (2000) was used to derive the required expressions in terms of the dimension of the ship for this purpose. The total time for a round-trip, which in effect determines the (variable) parameter NT above, is comprised of the times required for loading and unloading the cargo, the waiting time in port, and the time spent at sea. These factors all depend on the length, the beam, the depth, and the speed of the ship. In particular, the time for loading containers at a given port is a function of the number of cranes available. In contrast with previous studies, the number of cranes was formulated as a function of the length of the ship, and the resulting expression was made continuous through a linear response surface fit. Appendix A in Ganesan (2001) gives a complete mathematical form of this objective function.

We remark here that an alternative related measure of merit that is often examined (and sometimes even used as an objective function) is the return on investment. The *return on investment* (**ROI**), expressed in percentage per year, is defined as the ratio

$$\text{ROI} = \frac{\text{annual gross income} - \text{annualized total cost} + \text{salvage value of the ship}}{\text{invested capital}} \times 100\%.$$

The annual gross income is calculated by defining a suitable charge-rate (CR), expressed in dollars per metric ton shipped per nautical mile (user-defined as 0.0064). The salvage value is taken as five percent of the depreciated value of the total investment, which is a reasonable value, assuming that the ship is in operable condition throughout its expected life-time. The ship life is identified by the user, and is taken as 20 years in this work. Accordingly, we can write

$$\text{ROI} = \frac{(\text{CR} \times \text{Wcargo} \times \text{Dst}) - \text{AC} + \left[\dfrac{0.05 \times \text{Owc}}{(1 + Ir)^{S\ell}}\right]}{\text{Owc}} \times 100\% \tag{15}$$

where Owc = cost of the ship to the owner in dollars, Ir = interest rate expressed in percentage/100 (user-defined as 0.08), and $S\ell$ = ship life in years (user-defined as 20), and where the remaining quantities are as defined previously. Appendix E in Ganesan (2001) provides a complete mathematical form of this economic index.

The resulting containership design problem, CSD, can be stated in the following generic form, similar to (1).

$$\textbf{CSD:}\quad \text{Minimize}\quad f(x) \tag{16a}$$

$$\text{subject to}\quad g_1(x) = 0 \tag{16b}$$

$$g_i(x) \geqslant 0 \qquad \forall \quad i = 2, \ldots, 4 \tag{16c}$$

$$h(x) \geqslant 0 \tag{16d}$$

$$x \in \Omega^0 \left\{x : \ell_j^0 \leqslant x_j \leqslant u_j^0, \quad \forall\ j = 1, \ldots, n \equiv 5\right\}. \tag{16e}$$

Here, the objective function $f(x)$ is the freight rate required to break-even as given by (14), the equality constraint (16b) represents the displacement and weight balance restriction, the inequalities in (16c) are respectively given by (9), (10), and (12), the separately highlighted inequality (16d) represents the linear constraint (7), and (16e) delineates the initial lower and upper bounds on the design variables as specified by (13). Note that as mentioned previously, the design variables x_j, $j = 1, \ldots, 5$, are assumed to be indexed in the order of diminishing priority with respect to their relative importance in the model, as reflected by the evident sensitivity of the objective and constraint functions to their selected values. To obtain these priority indices for the design variables, the effect of each design variable on the objective and constraint functions was studied. This was done by examining the objective and constraint functions as functions of each design variable, defined over its bounding interval, while the other design variables were kept fixed at certain nominal values within their respective ranges. (Other than this, any further cross-interaction effects were ignored in this analysis for determining variable priorities. Note that as the algorithm proceeds and more information becomes available, these priorities could be altered at a later stage within the algorithmic process.) Linear response surface fits were then constructed for the nonlinear objective and constraint functions over each separate dimension of the hyperrectangle of interest defined in (16e). For functions exhibiting a strict local minimum in the range, the design space was partitioned into two subregions, one on each side of the strict local minimum, and linear response surface fits were constructed for both regions. The slopes of the linear response surface fits were then computed. (For functions where partitioned subregions of the design space were constructed, the average absolute value of the slope was used.) The (absolute) resulting slopes of the objective and constraint functions with respect to each design variable were averaged, and these values were then ranked in decreasing order to determine the relative importance, or the priority index, of the design variables in the model.

4. Computational Results

In this section, we present computational results pertaining to applying the pro-

Table 1. Summary of the response surface approximations for the initial root-node

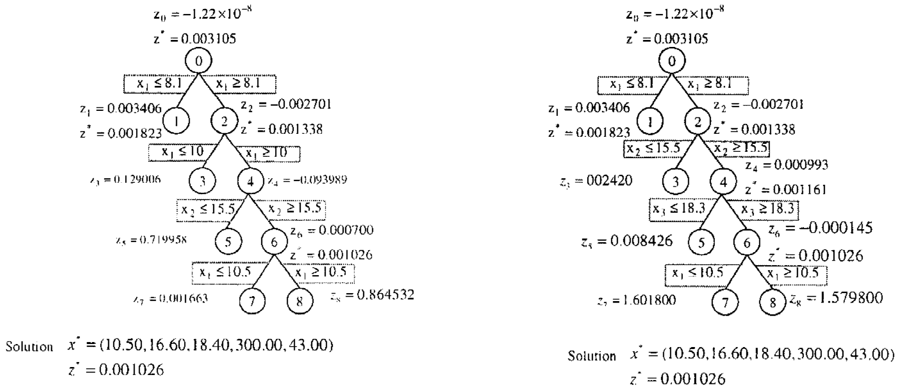
Response function	Design of experiment points		Significant response terms incorporated	R^2 statistic
	Initial full-factorial design	Geometrically feasible points		
$f(x)$	1024	706	117	0.9941
$g_1(x)$	1024	706	134	0.9984
$g_2(x)$	1024	706	133	0.9971
$g_3(x)$	1024	434	29	0.9998
$g_4(x)$	625	536	150	0.9661

posed pseudo-global optimization approaches to a typical test case of a container-ship design problem CSD using practical data as specified in Neu et al. (2000). The following RSM stepwise strategy was employed to construct the required polynomial approximation response surfaces.

1. *Perform a screening using a full factorial experimental design* (*see Myers, 1995*). The objective function $f(x)$, the constraint function $g_1(x)$ that equates the weight and the displacement, and the functions $g_2(x)$ and $g_4(x)$ that define the inequality constraints on the metracentric height and the rolling period, respectively, are each defined in terms of all the five decision variables. For these functions, we employed a full factorial design having four levels, leading to $4^5 = 1024$ points. On the other hand, the function $g_3(x)$ that defines the inequality constraint on the freeboard is described in terms of four decision variables. For this function, we employed a full factorial design having five levels, leading to $5^4 = 625$ points. (The resulting fifth-order polynomial approximations for these functions contain at most 252 terms and 126 terms, respectively, several of which turned out to have zero (statistically insignificant) coefficients.)
2. *Apply the geometric constraint on the length to depth ratio as described in (7) to eliminate the experimental points that correspond to infeasible designs*.
3. *Perform a regression analysis to obtain the coefficients of the fitted polynomial response functions of order d =5*. (We used a forward stepwise regression process using the *JMP* software obtained from SAS Institute Inc. (2001), with threshold values of 0.25 and 0.10 for terms to enter and leave the model, respectively, as determined by statistical tests.)

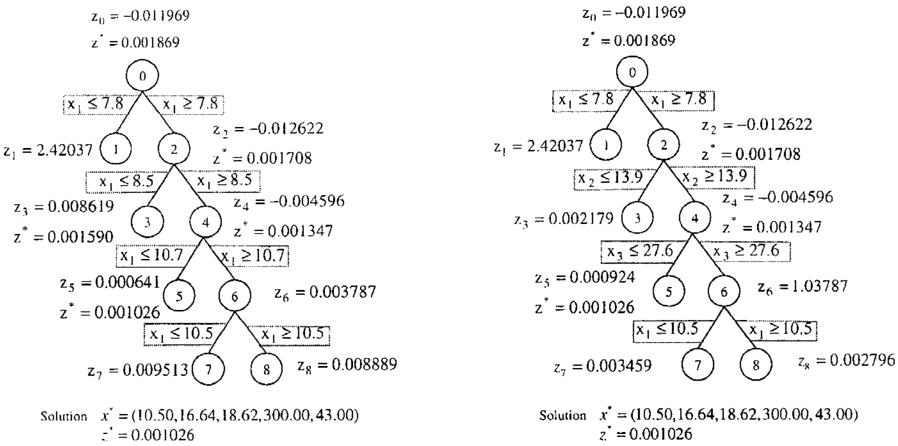
This process was verified to yield very accurate representations. For example, for the initial root-node, where the bounding hyperrectangle is given by (13), a summary of the response surface approximations is provided in Table 1.

Figure 3 presents the results obtained for Algorithm PGO1 using the two alternative branching rules BR1 and BR2, and Figure 4 presents analogous results for Algorithm PGO2. In these figures, the value of the incumbent solution z^* is indicated only at those nodes where this value was updated. Note that Algorithm



(a) Algorithm PGO1-BR1 **(b) Algorithm PGO1-BR2**

Figure 3. Results for algorithm PGO1.



(a) Algorithm PGO2-BR1 **(b) Algorithm PGO2-BR2**

Figure 4. Results for algorithm PGO2.

PGO1 produced the same solution using either branching strategy BR1 or BR2, as did Algorithm PGO2. The two solutions produced by these algorithms differ only slightly but have the same objective value (to six decimal places). Table 2 displays the principal ship design parameters corresponding to these two solutions. The analysis of each node for PGO1 consumed about 2183 and 42 cpu seconds for the lower and upper bound computations, respectively, while these run-times for PGO2 were about 40 and 48 cpu seconds, respectively, on a SUN Ultra 1 workstation operating Solaris 2.5.1. Note that the total time for PGO2 was only about 4% of that consumed by PGO1 because of the simpler linear programming lower bounding scheme, along with the fact that it required the enumeration of the same number of nodes in this case.

Table 2. Design parameters obtained from algorithms PGO1 and PGO2

Containership design parameter	Algorithm PGO1	Algorithm PGO2	Neu et al. (2000)
Carrying capacity (TEUs)	5,716	5,713	3,591
Number of roundtrips made annually	17.06	17.21	16.58
Annual transportation capacity (TEUs)	97,515	98,321	59,539
Annual dead weight transported (metric tons)	1,170,179	1,179,849	714,465
Annual gross weight transported (metric tons)	1,593,249	1,593,474	976,494
Cost to owner	$54,334,781	$54,403,474	$39,968,976
Annual profits for full-load shipments	$44,019,813	$44,383,551	$25,756,476
Return on investment	80.83%	83.55%	64.58%

The design determined by Algorithm PGO2 has a slightly greater depth and the service speed is somewhat greater. Although the increase in depth is not significant enough to cause an increase in the carrying capacity, the faster speed does enable it to log additional nautical miles annually, and this is also reflected in the improved return on investment as shown in Table 2. For both these solutions, the only active constraint is the mandatory equality restriction that balances the displacement and the weight. A sensitivity analysis performed in the local neighborhood of these solutions reveals that the objective function is relatively flat, indicating that the user has some nominal freedom in modifying the design parameters.

For the purpose of comparison, we also applied the design optimization tool developed by Neu et al. (2000) to solve this instance of Problem CSD. This tool employs three nonlinear search methods: the Modified Method of Feasible Directions (MMFD), Sequential Linear Programming (SLP), and Sequential Quadratic Programming (SQP) (see Bazaraa et al., 1993, for a description of these algorithms). The methodology used by Neu et al. first applies each of these methods to three starting solutions as determined by the lower bounds, the upper bounds, and the interval midpoints of the hyperrectangle Ω^0. Subsequently, using the best solution thus obtained for each method as a new starting solution, the procedure re-runs each of the three methods with certain prescribed tolerance and parameter settings. The best overall solution found by these multiple (twelve) runs for our model data is given by $x^* = (10.00, 17.00, 16.70, 211.19, 40.90)$ and has an objective value of $z^* = 0.001250$, with some additional design and performance measures being specified in Table 2. Note that the Algorithms PGO1 and PGO2 result in a 17.92% improvement in the objective function value. Recall that the objective function used is the required freight rate to break-even expressed in dollars per metric ton per nautical mile. Hence, this results in an estimated increase in profits of about $18.45 million over the life of the ship. Moreover, at an average, the return on investment over the life of the ship for the designs produced by the proposed algorithms improves on that realized by the Neu et al.'s design by a factor of about 27%.

5. Summary, Conclusions, and Extensions

In this paper, we have presented a class of pseudo-global optimization approaches for solving challenging optimization problems that are defined in terms of complex functions that are expensive to evaluate, or in terms of black-box functions. Such problems arise frequently in engineering design and process control contexts. We have described a methodology based on using RSM or interpolation techniques to construct polynomial programming approximations, and then using these in concert with global optimization methods such as the Reformulation-Linearization Technique (RLT) along with suitable partitioning schemes within a branch-and-bound framework. To illustrate this methodology, we have applied it to a detailed improved model developed for the design of containerships. A comparison of the design obtained by using the proposed approach with that resulting from the application of the incumbent nonlinear design optimization tool of Neu et al. (2000) revealed a significant improvement in the design parameters, translating to an estimated increase in profits of about $18.45 million, and an estimated 27% increase in the return on investment, over the life of the ship. For future research, we recommend a more detailed study of the convergence characteristics of the proposed class of pseudo-global optimization methods, and an automation of its application to various design problems. Note that in our computational runs, because of the prototypical nature of our procedure that employs disparate tools for constructing response surface approximations, solving polynomial programming approximations, and performing local searches, each node analysis required an intense manual interaction for implementing the prescribed steps. It is worthwhile to point out here that available statistical software do not always allow the user to construct response surface models of orders greater than quadratic, and the ones that do allow this, require the additional polynomial terms to be added manually. Hence, the automation required is a nontrivial task, but one that is necessary to test the proposed methodology using additional case studies, as well as to facilitate the transfer of this technology to practice.

In conclusion, we mention that the developed approach can also be extended to the design of other vehicles and structures. One such possible application is the design of surface warships. This problem is somewhat more intricate because it additionally requires the consideration of issues such as *susceptibility* (the ability to avoid detection), and *vulnerability* (the ability to withstand damage in action). Depending on the type of warship (aircraft carrier, frigate, destroyer, or minesweeper), the design variables would relate to the principal dimensions and the coefficients of form similar to the containership design problem described in this work, but would also include other design considerations such as the design speed in calm water, cruise speed at which the specified level of endurance is to be attained in calm water, and navigable distance at the cruise speed with all available fuel used. Constraints would need to be imposed on the maximum permissible operational load, the minimum number of watertight bulkheads, and the minimum number of decks below the upper deck, in addition to the constraints pertaining to the stability, the rolling period, and the minimum required freeboard. A possible objective function could be to maximize the transport effectiveness that is a combination of the specific power and the ratio of the operational load to the total weight of the ship. Brown and Tupper (1989), Hovgaard (1920), Rawson and Tupper (1984), and Taggart (1980) provide further guidelines and more detailed discussion on these design issues.

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